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## Distributions of Birth Intervals Based on Original Data''

### Introduction

FERTILITY is determined by an elaborate complex of diverse factors— biological, psychological, socio-economic, anthropological and others. Its level is the outcome of the interplay of these factors. Hence, a proper study of fertility can be done only on the basis of original and reliable data.

The data for this article are taken from the Kerala Standard Fertility Survey conducted in 1965. The main objectives of the study were the following: (i) To assess the current levels of fertility and to measure the changes in fertility which occur from time to time in local areas, and (ii) to collect data for testing the relative sensitivity of the various indices that can be used to detect changes in fertility behaviour.

The survey was conducted in three panchayats (local administrative units each comprising of several villages) near Trivandrum city. An intensive family planning communication and education programme has been going on among

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the target population since 1962. The majority of the people of this rural area belong to the lower economic strata which is typical of most of the villages in Kerala. The sample for the study was drawn from a complete list of households in the area. Seven independent systematic samples were drawn separately for each of the three panchayats so that the total households would come to around 2000. The sample size within each panchayat was proportional to its total size. The seven samples of each of the panchayats were allotted at random to the seven investigators. The investigators were to enumerate the same sub-sample at all revisits. Finally, after excluding non-dwelling units and unoccupied houses the total sample had 1994 houses and these houses had 2178 households eligible for the study.

Demographic and socio-economic information on each of the sample households was obtained from a responsible member of the household. The particulars pertaining to fertility were obtained directly, as far as possible, from each married woman of the household, who at the time of the survey was less than 51 years of age and was not permanently separated from her husband. Seven female investigators, who had sufficient experience in interviewing for demographic and socio-economic data, collected the relevant information. The response rate was 96.8 per cent. In this article the fertility aspect of the survey is dealt with.

Various authors have studied the distributions of inter-birth intervals. Throughout this article inter-birth interval refers to inter-live-birth interval. Sheps and Perrin (1964) considered the reproductive sequel as a Markov renewal process and developed expressions for moments of intervals between different states—fecundable state, pregnant state and post-partum amenorrhea state. Dandekar (1959), Henry (1958), Potter (1963) and Singh (1963) developed models assuming fecundability to be a constant for a group of women. Venkatacharya (1969) studied inter-birth intervals under various fertility parameters and marital durations. Jain (1969) examined some goodness-of-fit problems. Jain, Hsu, Freedman and Chang (1970) studied the demographic aspects of post-partum amenorrhea. In most of these studies discrete distributions are used whereas in this article continuous distributions, which are more suitable to the problems under consideration, are used. This is more appropriate because the length of the interval could be any non-negative number thus generating a continuum of points.

#### 4. Data on Inter-birth Interval

The following table gives the original data for parities zero to six. For convenience, the interval between the z'-th and the (z +1)-th live-births is taken as parity z and thus the interval between marriage and first live-birth is taken as parity zero.

TABLE 1—INTERVALS BETWEEN BIRTHS ACCORDING TO PARITIES

<i>Interbirth interval (in months)</i>	<i>Frequency = Number of Women in each interval</i>						
	<i>Parity = 0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>
<i>(1)</i>	<i>(2)</i>	<i>(3)</i>	<i>(4)</i>	<i>(5)</i>	<i>(6)</i>	<i>(7)</i>	<i>(8)</i>
0-9	19	—	—	—	—	—	—
9-12	44	1	2	0	3	1	1
12-15	27	13	5	2	6	5	0
15-18	15	16	11	13	5	7	3
18-21	12	13	8	4	6	7	8
21-24	18	19	18	17	13	8	10
24-27	11	18	26	21	20	12	16
27-30	9	20	18	22	12	7	11
30-33	5	18	16	17	14	16	7
33-36	5	18	19	15	11	18	9
36-39	5	16	19	14	17	12	12
39-42	3	15	13	11	11	8	2
42-45	4	9	13	7	12	6	5
45-48	3	11	8	4	4	3	6
48-51	3	9	7	9	8	4	6
51-54	5	5	3	7	10	7	1
54-57	0	3	1	5	3	2	1
57-60	0	1	6	1	4	0	0
60-63	1	3	4	3	0	3	1

Table 1 (contd. on page 166)

Table 1 (contd. from page 165)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
63-66	0	3	6	3	1	2	0
66-69	2	3	0	1	0	1	1
69-72	1	1	1	1	0	1	0
72-75	1	1	4	1	2	4	1
75-78	0	1	0	5	1	0	0
78-81	0	1	1	2	1	0	0
81-84	3	2	1	2	1	0	0
84-87	1	0	1	2	1	1	1
87-90	0	1	3	0	0	2	1
90-93	0	1	1	0	1	1	—
93-96	2	1	0	0	1	0	—
96-99	0	0	1	1	0	1	—
99-102	0	1	0	0	1	—	—
102-105	0	—	0	2	0	—	—
105-108	—	—	0	—	0	—	—
108-111	—	—	2	—	2	—	—
111-243	4	—	—	—	—	—	—
	203	224	218	192	171	139	103

From the data given in Table 1 it seems that a Gamma distribution is a good fit for the birth-interval in each parity. So an attempt is made to fit the following Gamma distribution:

$$f(x) = \frac{a^b}{\Gamma(b)} x^{b-1} e^{-ax} \quad \dots (2.1)$$

$$b > 0, a > 0, 0 \leq x \leq \infty,$$

where  $a$  and  $b$  are constants to be determined. It may be remarked that Brass (1959) has assumed a Gamma distribution for the expected rate of child-bearing among women and for some other uses of gamma distribution in demographic problems (see Sheps and Menken, 1973).

### 3. Estimation and Fit

There are different methods of estimating the parameters  $a$  and  $b$  in (2.1). The usual methods are the method of moments and the method of maximum likelihood. The estimating equations for the method of moments are the following:

$$\frac{b}{a} = m'_1, \frac{b(b+1)}{a^2} = m'_2 \quad \dots (3.1)$$

where  $b/a$  and  $[b(b+1)/a^2]$  are the first and second population moments about the origin and  $m'_1$  and  $m'_2$  are the corresponding sample moments. By using the data in Table 1 for parities 1 to 6 and the detailed data for parity zero the parameters  $a$  and  $b$  are estimated with the help of (3.1). These are given in Table 2. Other methods may lead to different estimates for  $a$  and  $b$ . Even

TABLE 2—ESTIMATES OF  $a$  AND  $b$  BY THE METHOD OF MOMENTS

Parity	Estimated $a$	Estimated $b$
0	0.0338	0.6418
1	0.1282	4.4698
2	0.1144	4.2884
3	0.1169	4.4454
4	0.1157	4.3493
5	0.1250	4.6663
6	0.1849	6.2891

if the underlying distribution is a Gamma still the parameters need not be the ones obtained in Table 2. These methods of estimation are suggested in statistical literature for different reasons. Each method gives estimates with different desirable properties. There is no unique set of desirable properties for estimates and there is no method of estimation which is universally good. Here the aim is to pick out the best fitting distribution to the given data. Hence the following procedure is used to guess the underlying distribution. Theoretical Gamma distribution is plotted for various values of  $a$  and  $b$ . Then these are superimposed over the empirical distributions of the various birth intervals. This method yielded the values of  $a$  and  $b$  for the theoretical Gamma distributions closer to the empirical distributions. These are denoted by  $A$  and  $B$ , Then

the fit of these distributions is tested with the help of standard goodness-of-fit tests. The above mentioned graphical procedure seems to be a very convenient tool in tackling similar problems.

Instead of using the above graphical method of determining the values of  $A$  and  $B$ , the following analytical procedure is suggested. This will also give a method of improving the estimates given by the method of moments—improvement in the sense of obtaining parameters, which give rise to distributions, which show a closer fit to the data.

From Table 2 it is seen that there is a possibility of a second degree polynomial relationship between  $a$  and  $b$  with the parities one to six. Hence by using the method of least squares, the following equations are obtained:

$$A = 0.007068 i^2 - 0.04049 i + 0.16539 \quad \dots (3.2)$$

$$B = 0.1725 i^2 - 0.9181 i + 5.3481. \quad \dots (3.3)$$

With the help of (3.2) and (3.3) improved estimates for  $a$  and  $b$  are obtained. These are given in Table 3.

TABLE 3—IMPROVED ESTIMATES FOR  $a$  AND  $b$  FOR PARITIES 1 TO 6

<i>Parity</i>	<i>Improved estimates of a</i>	<i>Improved estimates of b</i>
1	0.1291(0.1320)	4.41726(4.6025)
2	0.1156(0.1127)	4.38861(4.2019)
3	0.1115(0.1075)	4.40024(4.1463)
4	0.1168(0.1165)	4.52023(4.4357)
5	0.1315(0.1397)	4.54434(5.0701)
6	0.1555(0.1770)	4.67681(6.0495)

The entries in the brackets are the ones obtained by using regression lines (3.2) and (3.3). The other estimates are obtained by graphical methods. For testing the goodness-of-fit the estimates obtained by graphical method are used and thus the computations are based on these estimates. By using these improved estimates in Table 3 the Gamma distributions are fitted to the data. Thus,

for example, for parity 1 the density function

$$f(x) = \frac{(0.1291)^{4.4173}}{\Gamma(4.4173)} x^{3.4173} e^{-0.1291x} \quad \dots (3.4)$$

is fitted. The following diagram gives the empirical and theoretical distributions for parity 1.

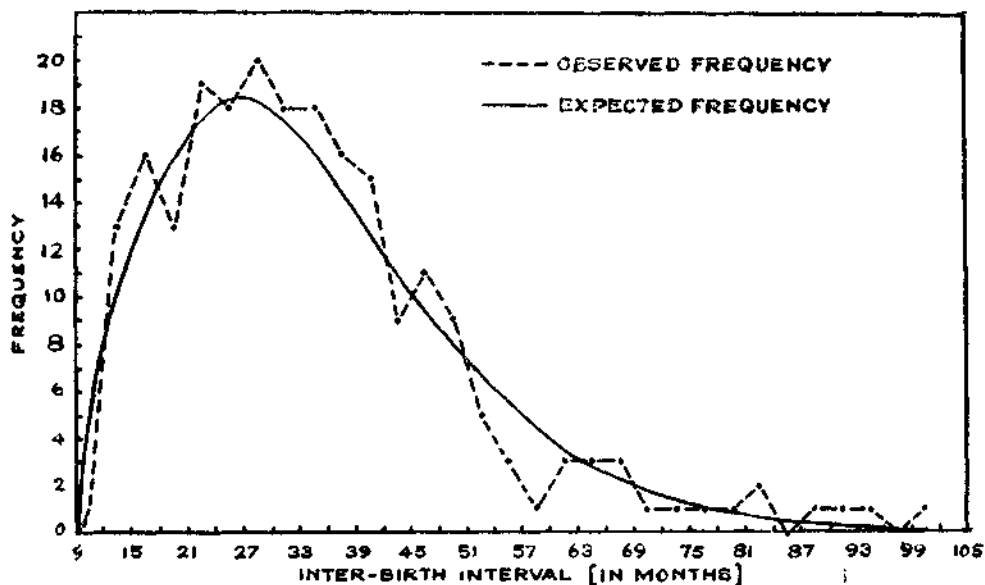


Fig. 1. Distribution of Inter-birth Interval (Parity-1).

From Figure 1 it is seen that the Gamma distribution is a good fit. This is also established by using a  $\chi^2$  goodness-of-fit test. The details of the calculations are given in Table 4.

Here we are taking the actual birth intervals. The birth interval for parity zero consists of the waiting time and the duration of pregnancy whereas in parities one and above the birth intervals consist of the post-partum amenorrhea, the waiting time and the duration of pregnancy. In these cases the duration of pregnancy can be assumed to be a constant equal to approximately nine calendar months or ten lunar months. Hence from Figure 1 we get only the shape of the Gamma distribution and the x-values do not start from zero. In

TABLE 4—CALCULATION FOR  $\chi^2$  GOODNESS-OF-FIT OF A GAMMA DISTRIBUTION FOR PARITY ONE

$x$	Observed frequency	Theoretical frequency	$x$	Observed frequency	Theoretical frequency
9-12	1	6.12109	57-60	1	4.38428
12-15	13	9.71221	60-63	3	3.53936
15-18	16	13.06975	63-66	3	2.83541
18-21	13	15.77159	66-69	3	2.25060
21-24	19	17.46588	69-72	1	1.77677
24-27	18	18.16862	72-75	1	1.39254
27-30	20	18.04629	75-78	1	1.07856
30-33	18	17.26137	78-81	1	0.83285
33-36	18	16.02895	81-84	2	0.64326
36-39	16	14.47463	84-87	0	0.49468
39-42	15	12.74542	87-90	1	0.40105
42-45	9	11.06036	90-93	1	0.26627
45-48	11	9.44147	93-96	1	0.21842
48-51	9	7.95500	96-99	0	0.16356
51-54	5	6.66346	99-102	1	0.12369
54-57	3	5.38816			

Mean = 34.8750,  $a = 0.1282$ ,  $A = 0.1291$ ,  $\chi^2_{14}$  (observed) = 14.5659

Variance = 272.1094,  $b = 4.4698$ ,  $B = 4.4173$ ,  $\chi^2_{14}$  (tabulated = 23.68 at 5% level)

Conclusion = Accepted the hypothesis

other words, if we shift the origin to the point  $x = 9$  we get the Gamma distribution for the birth intervals excluding the duration of pregnancy.

Evidently, for parities one and above the birth intervals thus obtained by subtracting 9 still contain the post-partum amenorrhea. Empirical studies indicate that this post-partum amenorrhea can be considered to be distributed as a truncated Gamma distribution. Thus the birth-intervals for parities one and above are of the form  $U = X + Y$  where  $X$  denotes the post-partum

amenorrhea and  $Y$ , the waiting time. Under the assumption of independence for  $X$  and  $F$  one can work out the distribution of  $U$ . The influence of  $X$  can affect the distribution of  $U$ . The empirical and theoretical Gamma distributions for parities 2 to 6 are given in Figures 2 (z) to 2 (v). From these figures it can be seen that the fit of a Gamma distribution becomes poorer and poorer as the parity number increases. This can be due to a multiplicity of reasons; a mathematical explanation for which is given towards the end of this article.

#### 4. Parity Zero

The usual assumption about waiting time is that it is exponentially distributed, see for example, Sheps and Menken (1973, Chapter 3). In our problem the interval for parity zero as already indicated consists of the waiting time, and the duration of pregnancy which can be assumed to be a constant equal to approximately nine calendar months or 10 lunar months. Hence, apart from a shift in location, the distribution of the interval for parity zero is the distribution of the

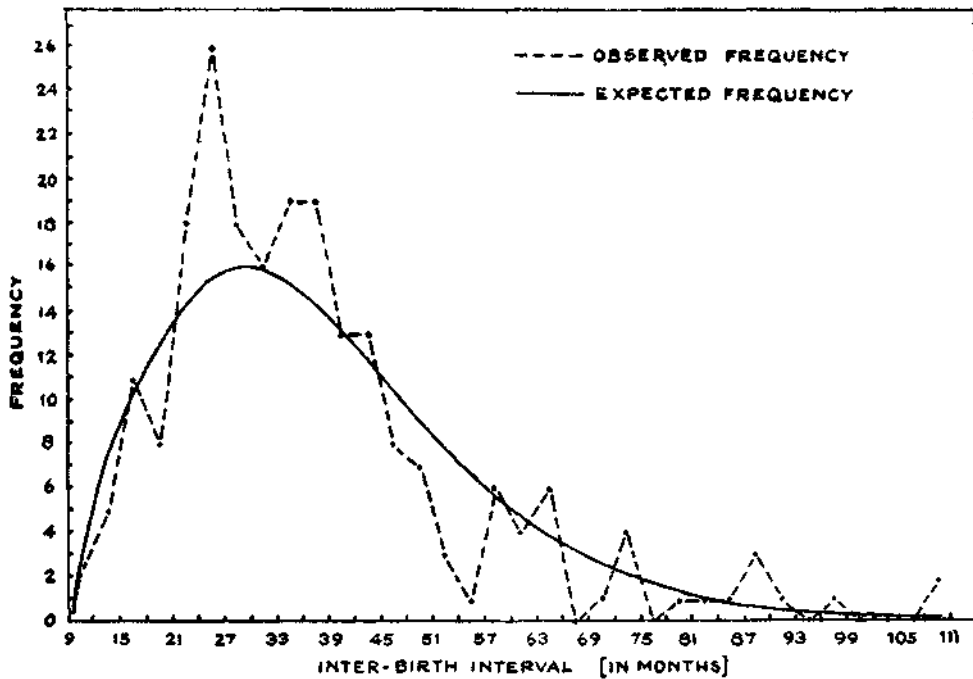


Fig. 2(i). Distribution of Inter-birth Interval (Parity-2).

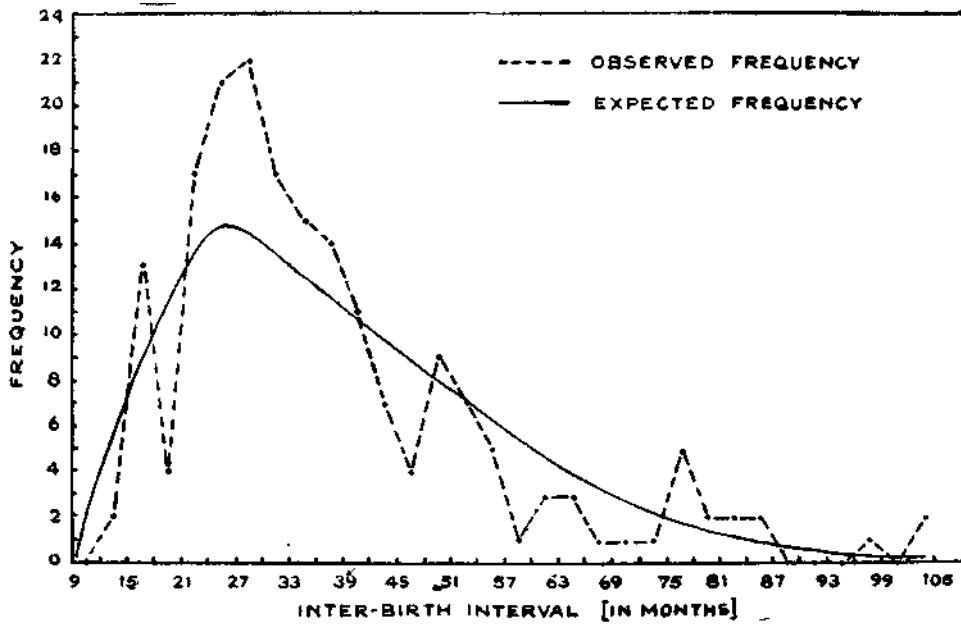


Fig. 2(ii). Distribution of Inter-birth Interval (Parity-3).

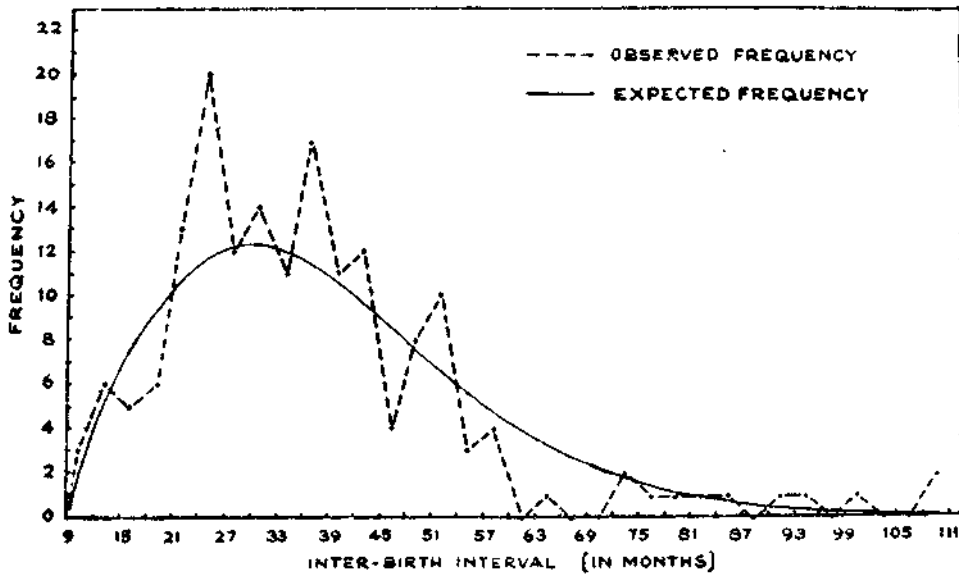


Fig. 2(iii). Distribution of Inter-birth Interval (Parity-4).

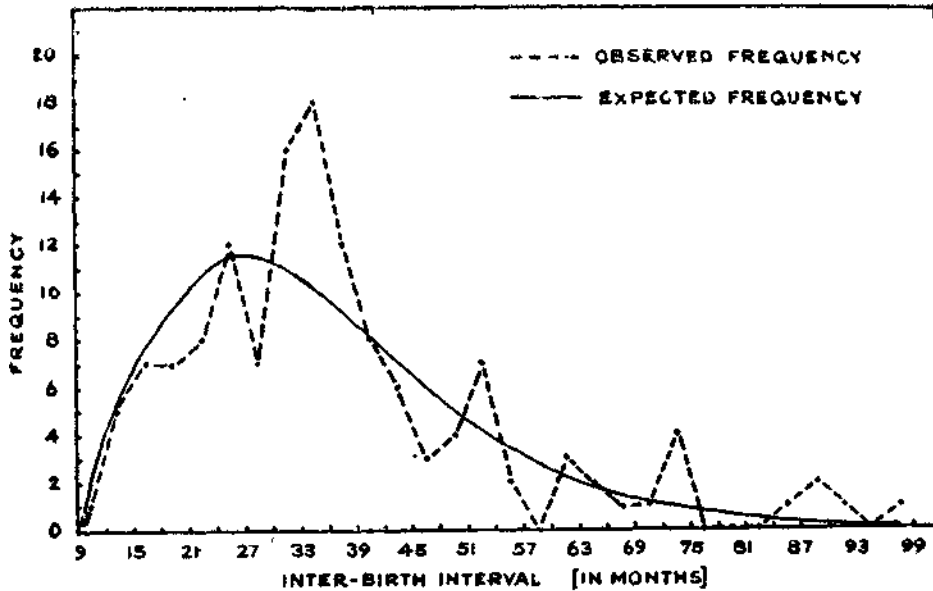


Fig. 2(iv). Distribution of Inter-birth Interval (Parity-5).

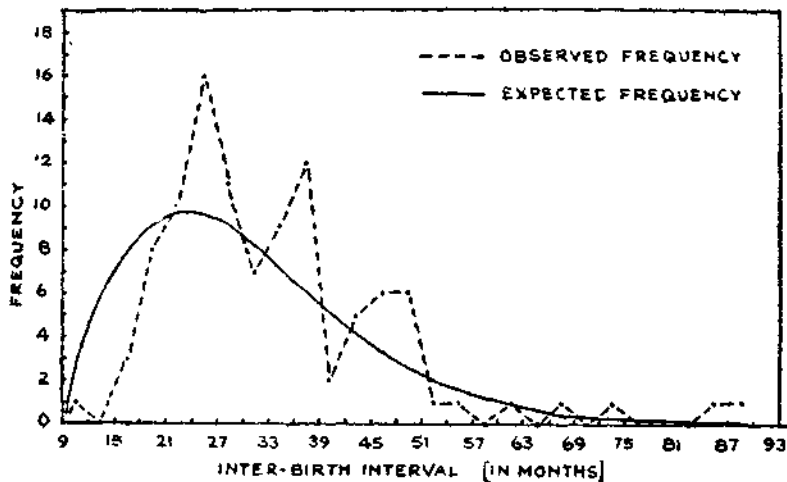


Fig. 2(v). Distribution of Inter-birth Interval (Parity-6).

waiting time itself. From Figure 3 and from the test of goodness-of-fit it is apparent that a Gamma distribution is a good fit. For mathematical convenience different authors assume exponentiality for the waiting time whereas this

assumption is questionable as far as birth-intervals are concerned. The exponential distribution is a limiting form of a geometric distribution. However, if we are ready to sacrifice some information and pool the frequencies in the

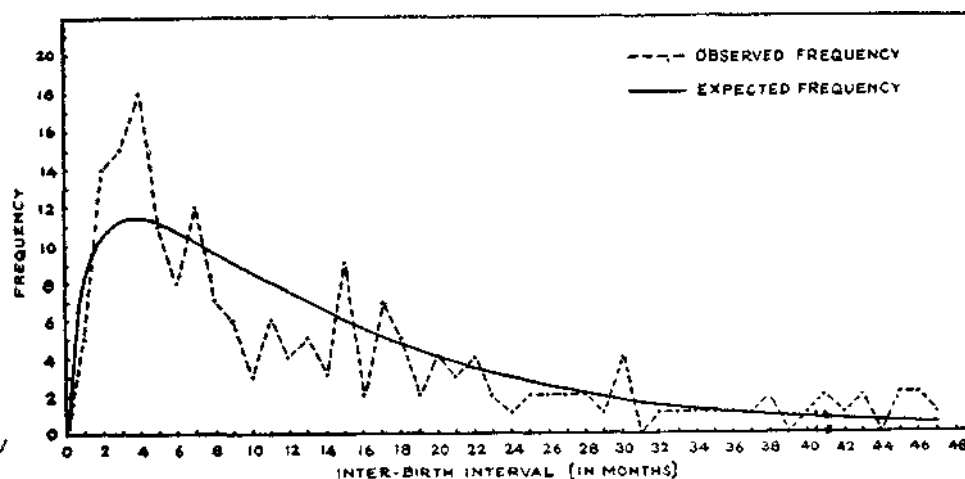


Fig. 3. Distribution of Inter-birth Interval (Parity-0).

beginning intervals then the empirical distribution will look like an exponential distribution. This may be one reason why authors mistake the underlying distribution to be exponential. Another reason is that if the probability of conception in any month is assumed to be a constant, say  $q$ , then the probability that the woman under consideration conceives in the  $r$ -th month is  $(1 - q)^{r-1}q$ . This probability, when approximated for the continuous case, leads to an exponential distribution under certain assumptions. Our empirical evidence indicates that this approximation to an exponential distribution is not a good one and that we lose information about the data by this approximation procedure. Also one can look at this assumption of exponentiality from a layman's point of view. Suppose that one assumes that the interval between the date of marriage and first pregnancy is exponentially distributed. This is in effect saying that the maximum number of pregnancies take place on the date of marriage or immediate to it which can be easily seen to be false by examining any real life data. For parity zero the data before classification contains some exceptional cases of women giving births 243 months after the date of marriage. These are shown in Table 1. Evidently these cases cannot be included while studying the distribution of waiting time among women in parity zero. The mean and variance given in Table 6 are the values obtained by using all the obser-

vations including the exceptional cases. But for fitting a Gamma distribution for parity one we have deleted the exceptional cases in the right tail. The data and the details of the calculations are given in Table 5 and a Gamma distribu-

TABLE 5—DETAILS OF THE CALCULATIONS FOR FITTING A GAMMA DISTRIBUTION FOR PARITY ZERO

<i>x-values</i>	<i>Observed frequency</i>	<i>Expected frequency</i>	<i>x-values</i>	<i>Observed frequency</i>	<i>Expected frequency</i>
1	5	8.2750	25	2	2.6429
2	14	10.4117	26	2	2.4324
3	15	11.1663	27	2	2.2370
4	18	11.2637	28	2	2.0292
5	11	11.0633	29	1	1.8600
6	8	10.6967	30	4	1.7104
7	12	10.2126	31	0	1.5718
8	7	9.6034	32	1	1.4436
9	6	9.0609	33	1	1.3251
10	3	8.5176	34	1	1.2156
11	6	7.9815	35	1	1.0961
12	4	7.4588	36	1	1.0063
13	5	6.9537	37	1	0.9235
14	3	6.4321	38	2	0.8471
15	9	5.9217	39	0	0.7766
16	2	5.4960	40	1	0.7116
17	7	5.0941	41	2	0.6497
18	5	4.7158	42	1	0.5867
19	2	4.3604	43	2	0.5380
20	4	4.0275	44	0	0.4930
21	3	3.6813	45	2	0.4516
22	4	3.3747	46	2	0.4136
23	2	3.1131	47	1	0.3785
24	1	2.8696	Total	188	199.0918

**Mean** = 13.4894, ***a*** = 0.0982,  $\chi^2_{21}$  (observed) 24.55

**Variance** = 137.3446, ***b*** = 1.3248,  $\chi^2_{21}$  (upper 5% tail) = 32.67

**Inference** = fit is good.

tion is seen to be a good fit. The x-values are taken after subtracting the duration for pregnancy so that the x-values will vary from zero onwards. This is done for convenience. Even otherwise the shape of the distribution can be seen from the graph except for a shift in the location.

The distribution that is fitted for parity zero is given by the density function

$$f(x) = \frac{(0.0982)^{1.3248}}{\Gamma(1.3248)} x^{0.3248} e^{-0.0982x} \quad \dots(4.1)$$

After deleting the exceptional cases and by using Table 5, it is seen that the mean birth interval is 13.4894 with a variance of 137.3446. The behaviour of these values is in agreement with the values in Table 6 for other parities. The expected frequencies in Table 5 are obtained by using linear interpolation. If double interpolation is used the values can be obtained slightly more accurately.

### 5. A Mathematical Explanation

When there is an effect due to post-partum amenorrhoea the birth interval for any parity from 1 and above will be of the form

$$U = X + Y \quad \dots(5.1)$$

where, as explained before, one can assume  $X$  to be a truncated Gamma variable and  $Y$  to be a Gamma variable. Let the density functions of  $X$  and  $Y$  be as follows :

$$f_1(x) = C x^{a_1-1} e^{-b_1x}, \quad 0 \leq x \leq k, \quad a_1 > 0, \quad b_1 > 0 \quad \dots(5.2)$$

$$f_2(y) = \frac{y^{a_2-1} e^{-b_2y}}{\Gamma(a_2)}, \quad 0 \leq y \leq \infty, \quad a_2 > 0, \quad b_2 > 0, \quad \dots(5.3)$$

where  $a_1, b_1, a_2, b_2$  and  $k$  are some constants and  $C$  is a normalizing constant for the density function  $f_1(x)$ . Under the assumption of independence for  $X$  and  $Y$  one can easily obtain the distribution of  $U$  by using known techniques. Let the density function of  $U$  be denoted by  $h(u)$ . Then

$$h(u) = \begin{cases} C_1 \int_0^u y^{a_1-1} e^{-b_1y} (u-y)^{a_2-1} e^{-b_2(u-y)} dy, & 0 \leq u \leq k, \\ C_2 \int_0^k y^{a_1-1} e^{-b_1y} (u-y)^{a_2-1} e^{-b_2(u-y)} dy, & k < u < \infty, \end{cases}$$

where

$$C_1 = \frac{C}{\Gamma(a_2)} \quad \dots (5.4)$$

That is,

$$h(u) = \begin{cases} C_1 e^{-b_2 u} u^{a_1+a_2-1} \int_0^1 t^{a_1-1} (1-t)^{a_2-1} e^{-(t_1-b_2)ut} dt, & 0 \leq u \leq k \\ C_1 e^{-b_2 u} k^{a_1} u^{a_2-1} \int_0^1 t^{a_1-1} \left(1 - \frac{kt}{u}\right)^{a_2-1} e^{-(b_1-b_2)kt} dt, & k < u < \infty \end{cases}$$

$$= \begin{cases} C_1 e^{-b_2 u} u^{a_1+a_2-1} \sum_{r=0}^{\infty} \frac{[(b_2 - b_1)u]^r}{r!} \frac{\Gamma(a_1 + r) \Gamma(a_2)}{\Gamma(a_1 + a_2 + r)}, & 0 < u < k \\ C_1 e^{-b_2 u} k^{a_1} u^{a_2-1} \sum_{r=0}^{\infty} \frac{[(b_2 - b_1)k]^r}{r!} \int_0^1 t^{a_1+r-1} \left(1 - \frac{kt}{u}\right)^{a_2-1} dt, & k < u < \infty \end{cases}$$

$$= \begin{cases} C_1 e^{-b_2 u} u^{a_1+a_2-1} \frac{\Gamma(a_2) \Gamma(a_1)}{\Gamma(a_1 + a_2)} {}_1F_1(a_1; a_2; (b_2 - b_1)u), & 0 \leq u \leq k \\ C_1 e^{-b_2 u} k^{a_1} u^{a_2-1} \sum_{r=0}^{\infty} \frac{[(b_2 - b_1)k]^r}{r!} \sum_{s=0}^{\infty} \frac{(1 - a_2) s (k/u)^s}{s!(a_1 + r + s)} & k < u < \infty \end{cases}$$

where  ${}_1F_1(\bullet)$  is a confluent hypergeometric function and in general the notation  $(a)_r$  means  $a(a+1)(a+2)\dots(a+r-1)$ .

The density  $h(u)$  approximates to a Gamma density and it will be a Gamma density when  $b_1 = b_2$  and when  $k$  tends to infinity. Thus for parity 1 a Gamma approximation may be obtained due to one of the above or other reasons. For parities 2 and above the fit is not that good. It may be due to the possibility of correlation with the variables associated with the earlier parities or it may

be that  $b_1$  is not equal to  $b_2$  or the underlying distributions may not be Gammas or due to other reasons. It must be remembered that if  $U = X + Y$  and if  $U$  is a Gamma and even if  $X$  and  $Y$  are independent, this does not characterize the distributions of  $X$  and  $Y$ . In other words, there are a number of possibilities for  $U$ . Also when  $X$  and  $Y$  are independent Gamma variables, the sum need not be a Gamma variable. So the discrepancies seen in parities 2 and above are to be examined in the light of the above observations and theoretical explanations.

Several other comments can also be made on the data in Table 1. For example, one can see that the frequencies decrease along the rows for higher values of  $x$ . In other words, women experience longer inter-pregnancy intervals at higher parities. This is also seen from the studies conducted by Jain, Hsu, Freedman and Chang on Taiwanese women. For example see Figure 2 in Jain, Hsu, Freedman and Chang (1970) which shows that older the women longer the post partum amenorrhea and thus longer the interval for the next parity. This interesting phenomenon is also seen mathematically from equations (3.2) and (3.3) where we have seen that there is a polynomial relationship existing between the parameters of the underlying Gamma distributions and the various parity numbers. That is, if the data in Table 1 are plotted on a surface then along the plane at parities 0 to 6 one gets the contours as Gamma distributions.

Another interesting property is seen from Table 6 which gives the mean birth intervals and variances for the various parities.

TABLE 6—MEAN VALUES AND VARIANCES OF BIRTH INTERVALS FOR DIFFERENT PARITIES

<i>Parity</i>	0	1	2	3	4	5	6
Mean Value	27.9754	34.8750	37.4862	38.0156	37.6053	37.3273	34.0048
Variance	561.0475	272.109	327.509	325.095	325.144	298.592	183.862

**Here** it may be noticed that the mean birth interval increases with parity 0 to 3 and then decreases for parities 3 to 6. The variances also behave more or less in the same fashion except for parity zero. In parity zero our data contained some exceptional cases. In a few cases it took up to 243 months

for the first conception. Evidently these observations are "outliers" in statistical terminology. Once these outliers are removed then the variance in parity zero will be considerably reduced and the behaviour of the mean and variances will fit in the general trend.

In this connection a minor comment is in order. In Jain, Hsu, Freedman and Chang (1970), the average post-partum amenorrhea increases steadily and it never comes down among Taiwanese women. The intervals under study in this article consist of the post-partum amenorrhea and the waiting time for the particular parity. Hence the behaviour of the intervals may not be too far away from the nature of the distribution of post partum amenorrhea. But our studies indicate that the mean intervals decrease after parity 3 rather than increasing steadily. This phenomenon may be interpreted in terms of psychological factors of anxiety among women about approaching menopause or it may be due to some other factors.

Here we will also list the details of the  $y^2$  values for the fit of Gamma distributions for the various parities so that one can see the nature of fit. The details of calculations are omitted in parities 2 to 6.

TABLE 7—THE  $\chi^2$  VALUES FOR THE GOODNESS-OF-FIT TESTS OF FITTING GAMMA DISTRIBUTIONS

<i>Parity</i>	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>
$\chi^2$ value observed	24.55 (21)	14.56 (14)	31.82 (18)	21.40 (12)	31.02 (16)	18.96 (13)	24.78 (10)
$\chi^2$ value at the upper tail	32.67 (5% level)	23.68 (5%)	34.81 (1%)	26.22 (1%)	32.00 (1%)	22.36 (5%)	23.21 (1%)
Inference	Accept the fit	Accept	Accept	Accept	Accept	Accept	Reject

The quantities given in the brackets are degrees of freedom. These degrees of freedom vary depending upon the groupings of the frequencies and the corresponding class intervals. It is noticed here that the fit in parity 6 is not a good one. The data in parity 6 is in fact the data for parities 6 or more. There are a few cases of higher parities in a sample. Hence we do not expect the Gamma distributions to be a good fit.

Graphical methods suggest that it may be possible to estimate the parameters of Gamma distributions which are still closer fits to the data for parities zero to five by using some simulation techniques.

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